Digital Being, the Real Continuum, the Rational and the Irrational

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1. Heidegger's move to a temporal meaning of being

> omnis singularis substantia agat sine intermissione, corpore ipso non excepto, in quo null unquam quies absoluta reperitur.[2]
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> Every singular substance acts without intermission, not excepting even the [physical] body, in which absolute rest is never to be found.

It is remarkable that today we are faced ubiquitously with digital technology — one could even say that it is 'in your face' — and yet the question concerning digital being has hardly surfaced in philosophical discourse. What computer science and information technology offer by way of 'ontologies' turns out to be simply various, more or less sophisticated taxonomies not worthy of the name 'ontology'. There is no way to raise the question concerning the digital cast of being that does not pass through Heidegger's thinking, for he is the one who resuscitated questioning of the very meaning of being. His opus magnum, *Sein und Zeit*, brings being into a relation with time, suggesting a temporal meaning of being. This move to temporality goes

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1 Paper for the *28th North Texas Heidegger Symposium* on 23-24 April 2010, not presented due to volcanic ash from Iceland's Eyjafjallajoekull over Europe and the consequent grounding of air traffic. The paper is based on my book *The Digital Cast of Being: Metaphysics, Mathematics, Cartesianism, Cybernetics, Capitalism, Communication* ontos verlag, Frankfurt 2009, available also with later postscripts in html at http://www.arte-fact.org/dgtlon_e.html

2 Gottfried Wilhelm Leibniz 'De Ipsa Natura sive de Vi Insita Actionibusque Creaturarum' (1698) *Philosophische Schriften* Band IV (ed.) Herbert Herring, Wissenschaftliche Buchgesellschaft, Darmstadt 1992 S. 288.

hand in hand with a momentous endeavour to repel what Heidegger calls the "invasion of the λόγος" already in the Greek beginnings of philosophy which, according to Heidegger, "is motivated by the fact that the ὄν, the being of beings itself, is interpreted primarily as presence and the λόγος is the way in which I primarily make something present".[3] The hegemony of logic today in all science and technology, and in their appendages, analytic philosophy and philosophy of science, thus has something to do with the unquestioned understanding of being as presence, and more specifically, as standing presence. Standing presence does not mean primarily enduring, permanent presence, but well-defined, logical presence to the mind at the present moment.

2. What is a digital being?

What does the Heideggerian questioning of the meaning of being have to do with those digital beings crowding today's world? What *is* a digital being, anyway? A first, short answer is that a digital being is composed of binary digits or bits, that is, of zeroes and ones, or of pure, well-defined difference. A digital bit can be a zero or one written on a piece of paper or, more often, one of two well-defined states of a material medium, such as the magnetic orientation of iron molecules.

A digital being consists of a finite sequence of bits which is, moreover, a pro-gram, that is, a pre-script or piece of software which is interpreted by a corresponding piece of hardware to bring about a foreseen change, such as the display of text on a screen, or a finished, turned table leg. Software and hardware fit together like the two pieces of a σύμβολον in the Greek sense. The software is binary program code

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[3] "*Dieser Einbruch des* λόγος, des Logischen in diesem streng griechischen Sinn, *in diese Fragestellung nach dem* ὄν *ist dadurch motiviert, daß das* ὄν, *das Sein des Seienden selbst, primär als Anwesenheit interpretiert ist und der* λόγος *die Art ist, in der ich mir etwas,* nämlich das, worüber ich spreche, *primär vergegenwärtige*." M. Heidegger *Platon: Sophistes* Marburger Vorlesung WS 1924/25 *Gesamtausgabe* Band 19 ed. Ingeborg Schüßler, Klostermann, Frankfurt/M. 1992 S. 225 = GA19:225, italic emphases by Heidegger himself.

inscribed in an electromagnetic medium, and the hardware can be any sort of machine fitted with a digital processor that is able to interpret the digital code as a machine command to bring about a foreseen, precalculated change. The machine interprets both digital program code and also data fed into it, which may be a text or sensory data of whatever kind, which themselves must first be converted into bits in order to be machine-interpretable. An example is a digital camera, which is equipped both with a digital program and is also fed quantified light data that it is able to convert into an image.

3. The outsourcing of the logos in digital technology

So much for an extremely cursory description of what a digital being is. This has everything to do with what digital technology is able to do today, but what does it have to do with the meaning of being as investigated and questioned by Heidegger? A clue is provided by the word, 'technology', the λόγος of τέχνη. Whereas in artisanal production, the technical know-how of, say, carpentry, is embodied in the carpenter who is thus able to convert wood into a table, with digital code, the know-how of carpentry can be encoded in software which is then embedded in a machine such as a lathe used to produce tables. Digital code is therefore a pre-script composed of binary digits embodying technical know-how of how to bring about or *produce* a foreseen, desired change of whatever kind.

The unique hallmark of specifically *digital* technology is that it is binarily encoded productive understanding of a segment of the world *outsourced* to an electromagnetic medium to control a machine. With the outsourcing of technological know-how as digital code inscribed in a machine's digital processor, *cybernetics* is born. Outsourced cybernetic code works independently of the programmer who wrote the code, giving rise to *automated systems* and *robots* which, depending on the intricacy and nesting of myriad routines of binary code, become more and more complex and eerily autonomous, even to the point of producing unforeseen, unwanted or disastrous outcomes.

The logical digital code encoding technical know-how is the *standing presence* whence movement in the world can be controlled. This fits the

description of δύναμις μετὰ λόγου or "power guided by the logos" that Aristotle takes as the paradigm for unfolding the ontology of movement or κίνησις as developed in Book Theta of his *Metaphysics*. Heidegger, in turn, subjected this Aristotelean ontology of movement to detailed phenomenological scrutiny in various lecture courses and writings.[4] Δύναμις μετὰ λόγου means a productive power over movement guided by a logical understanding of the world.

4. Greek ontology of movement

The question concerning the being of movement was the great problem of ancient Greek philosophical thinking. Plato's famous dialectic of ideas in *The Sophist* involves five categories: being, the self-same, the other, standstill and movement whose upshot is a sixth quasi-category, namely, non-being (μὴ ὄν), which Parmenides had denied. This is the closest Plato comes to solving the riddle of the ontology of movement of all kinds on the basis of the tacit Greek understanding of being as standing presence. For Plato, it is above all the ἰδέα, the εἶδος that has standing presence.

At the acme of ancient Greek philosophy, Aristotle takes Plato one step further in coming to terms with the ontology of movement. Aristotle's deep insight into the peculiarity of the phenomenon of movement and change is that anything in movement has a *twofold* (διχῶς) presence: first of all it shows itself in the look of its εἶδος, but secondly, it also has a lack (στέρησις) that points to something absent

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4 Cf. M. Heidegger *Grundbegriffe der aristotelischen Philosophie* Marburger Vorlesung SS 1924 *Gesamtausgabe* Band 18, ed. Mark Michalski Klostermann, Frankfurt/M. 2002 § 26. Bewegung als ἐντελέχεια τοῦ δυνάμει ὄντος (*Phys*. Γ 1) et seq.; M. Heidegger *Die Grundprobleme der Phänomenologie* Marburger Vorlesung SS 1927 ed. F-W. v. Herrmann 1975 § 19 a) β) Auslegung des Aristotelischen Zeitbegriffs; M. Heidegger *Aristoteles, Metaphysik Θ 1-3: Von Wesen und Wirklichkeit der Kraft* Freiburger Vorlesung SS 1931 *Gesamtausgabe* Band 33, ed. Heinrich Hüni 1981; M. Heidegger 'Vom Wesen und Begriff der φύσις: Aristoteles, *Physik* B, 1' (1939) in *Wegmarken* Klostermann, Frankfurt/M. 2nd. ed.1978.

which it could also *be*, i.e. which also could come or be brought into presence. For a moving, changing being, an absence is present. For instance, a piece of timber presents itself in its εἶδος as timber and also as lacking what it could also be, namely, a table, for instance. The thing itself has an *inherent* tendency to become other than it is; it is not yet finished.

Aristotle conceives the lack in the twofold presence of a being in movement through the pair of concepts, δύναμις and ἐντελέχεια. A being with a potential, a δυνάμει ὄν, has the power to become something else, but as it is in its presence, it is still ἀτελής, unfinished. It could only *have* itself in its finished presence in achieving ἐντελέχεια, i.e. through its having-itself-in-its-end. Thus does Aristotle come to his first definition of the being of movement. It is the presence of the potential being *as such*, stretching itself toward its finished presence, *underway* toward becoming other than it is, in a finished state in which the movement will have come into its end. In movement, the being's power to be what it *can* be is *at work*, i.e. it is ἐνέργεια. Movement itself is *both* presence and absence and must be addressed by a pair of ontological concepts, δύναμις and ἐντελέχεια as lack (στέρησις), whose unified twofold presence is a third phenomenon, namely, the at-work-ness of the potential *underway* toward finished presence.

In its indispensable basic concepts of force, power, work and energy, modern physics still employs the Aristotelean ontology of movement. The illogical nature of movement of all kinds as a simultaneity of presence and absence continues to haunt modern physics, turning up in unexpected places such as quantum mechanics in the guise of Heisenbergian uncertainty. In its ontological blindness, modern physics futilely tries to come to terms with the strange paradoxes of quantum mechanics by means of sophisticated experiments that are supposed to either confirm or refute mathematico-theoretical constructions. What has to be posed, however, is a genuine *ontological* question concerning the as-yet unquestioned implicit understanding of being as standing presence.

5. Aristotelean ontology of time

Having brought digital cybernetics into a connection with the Aristotelean ontology of movement, allows us to proceed to the question of time lurking behind any consideration of movement and change, and in particular, the modern cybernetic will to power over movement of all kinds. For Aristotle, time is an ἀριθμός or number abstracted from movement. Time is for him, and thereafter for Western science, an abstraction. A number for the ancient Greeks is first and foremost a counting number and secondarily fractions of counting numbers, so-called positive rational numbers.

Time for the Greeks is thus a number counted off movement. Most often this movement is taken to be the regular, circular motion of the stars in the skies. Since this celestial motion is uniform, it can also be subdivided into convenient smaller units such as hours and minutes by means of the angular measurements of definite fixed stars in the night skies or angular measurements during the day on a sun dial. From these basic celestial motions, any movement or change on Earth can then be counted in a convenient unit such as days or hours or minutes. What is counted is always a steady drumbeat of nows in the present. Time itself is therefore conceived from counted instants, which are the standing presence of time. The unit of time may be broken down into very small units by human artifice, giving rise to pendulum clocks, wind-up clocks, through to incredibly accurate modern atomic clocks. Despite increasing accuracy, clock-time is always a counted time based on a unit that is a fraction of a natural motion which today is taken to be ephemeris time. Since clock-time is counted time abstracted from some physical motion or other, it is firstly *quantitative* and secondly, necessarily *discrete*.

The quantitative nature of clock-time means that it is simply a counted number-residue amenable to arithmetic calculation. The discrete nature of counted clock-time, however, leads to a dilemma, for it is read off movement, which is continuous. The continuity of movement is therefore lost in abstracting to discrete, counted time, no matter how fine the units of clock-time become.

Greek mathematics knew of no way[5] to bring together arithmetic, which deals with sequential discrete numbers, and geometry, which deals with continuous figures. Greek mathematics' attempt to bring arithmetic and geometry together led to the discovery of the irrational numbers, which are irrational because they can be counted neither by the natural counting numbers, nor by rational fractions, no matter how small they may become. When counting and fractional numbers are applied to geometrical figures there are always magnitudes that cannot be brought into the form of a rational fraction. The simplest example is the length of the hypotenuse of a right-angled, isosceles triangle. The irrational numbers in the geometrical continuum thus came to be called surds, or absurd numbers.

The irrational numbers beyond any counting process but nevertheless present in any continuity give rise to considerable dilemmas in the effort to think through the nature of clock-time, for clock-time is a forever *discrete*, countable number lifted off a *continuous* movement. As a countable number, there are always gaps in clock-time, namely irrational numbers that can never, ever be *brought to presence* in any counting process, no matter how long it may proceed. It can easily be shown mathematically that between any two rational numbers whatsoever, no matter how close together they may be, there is always an uncountable irrational number. It is therefore impossible for continuous movement of any kind to take place in discrete clock-time, which is a remarkable paradox for physics, dealing as it does with movable beings. Even today, in the most advanced mathematical physics, the motions of physical beings are situated in a mathematically construed space-time, for which the antinomy between continuous time and discrete clock-time is played down or rather, entirely overlooked.

Some twentieth century physicists, such as the renowned John Wheeler,[6] nevertheless recognized the great challenge of coming to

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5 Cf. Jacob Klein *Greek Mathematical Thought and the Origin of Algebra* transl. Eva Brann, Dover Publications, New York 1992, first published by M.I.T. Press, Cambridge Mass., 1968 p. 193.

6 Cf. John Archibald Wheeler 'Hermann Weyl and the Unity of Knowledge' *American Scientist* Vol. 74, July-August 1986, pp. 366-375. Adapted from

terms with time in theoretical physics, and other physicists, such as Julian Barbour,[7] in a kind of neo-Parmenidean move, today propose doing away with time altogether in advanced quantum gravity theory. If time and space themselves are conceived as ultimately discrete, bounded by the Planck constant, as some physical theories propose today,[8] how is motion at all possible? To this day, modern physics does not entertain a single thought in the direction of thinking time itself phenomenologically as three-dimensional.

6. The disjuncture between discreteness and continuity and its overcoming in modern mathematics under Descartes' *Rules*

We should note with respect to the digital beings at the focus of attention here, that they themselves are composed of discrete numbers but are employed cybernetically to control movements, so, here too, there is a disjuncture between discreteness and continuity.[9] Waves, including electromagnetic waves, are conceived as continuous, whereas streams of particles are necessarily countable and discrete. Since Planck and Einstein and the founding of quantum physics, the wave-particle duality of sub-atomic entities has become almost a commonplace, albeit ontologically still poorly understood.

W. Deppert (ed.) *Proceedings of the Internationaler Hermann-Weyl-Kongress: Exakte Wissenschaften und ihre Philosophische Grundlegung* (Peter Land), 1986, available at www.weylmann.com

7 Cf. Julian Barbour *The End of Time* Weidenfeld & Nicolson, London, and Oxford University Press, New York 1999 and his shorter essay, 'The Nature of Time', available at http://www.platonia.com

8 Cf. Joy Christian 'Absolute Being vs Relative Becoming' in *Relativity and the Dimensionality of the World* within the series *Fundamental Theories of Physics* edited by Vesselin Petkov, Springer, NY 2007, available at http://arxiv.org/abs/gr-qc/0610049v2, accessed August 2009.

9 For more on this disjuncture, see 'Postscript 1: On the antinomy between countable discreteness and the continuum in twentieth-century mathematical foundations (Solomon Feferman and Hermann Weyl)' to my *Digital Cast of Being* op. cit., available at http://www.arte-fact.org/dgtlon_e.html#ps1

An obvious objection to the considerations just raised is that mathematics since the Greeks has made enormous strides far beyond the horizons of Greek philosophical thinking and thus that these quaint dilemmas between discreteness and continuity have been resolved. There is some apparent truth in this, so let's take a closer look. There are two aspects to consider. One is the development of mathematics since the seventeenth century, and the other is the closely related, explicit inauguration of the Cartesian mathematical casting of being in the same century. To take the latter first, Descartes' *Regulae* or *Rules* read like a blueprint for the mathematical, scientific method of access to being that lays the foundation for the modern age. Rule XIV.4 prescribes that the being of beings has to be "comprehended under the term 'magnitude'" which admits "a more or less",[10] thus enabling a reduction "in such a way that the equality between what is sought and something known becomes clearly visible" (XIV.3).[11]

Abstracting from the phenomena to obtain a magnitude means approaching, or rather interrogating, all phenomena under the dictate of measurability in order to allow equations to be formed between what is known and what is unknown. Descartes' Rule XII.11 prescribes that "things themselves are not to be laid before the external senses, but rather certain abbreviating figures"[12] which can be further compacted into "the briefest of signs" (brevissimas notas; Rule XVI). These briefest of signs are what we understand today as algebraic variables and

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10 More fully: "It is to be noted finally that nothing can be reduced to this equality if it does not admit a more or less and that all this is to be comprehended under the term 'magnitude' so that [...] we understand that from here on we are involved only with magnitudes in general" (Notandum est deinde, nihil ad istam aequalitatem reduci posse, nisi quod recipit majus et minus, atque illud omne per magnitudinis vocabulum comprehendi, adeo ut [...] hic tantum deinceps circa magnitudines in genere intelligamus nos versari, R. Descartes *Regulae ad Directionem Ingenii* Philosophische Schriften Meiner, Hamburg, 1996. XIV.4).

11 "in proportionibus istis eo reducendis, ut aequalitas inter quaesitum, et aliquid quod sit cognitum, clare videatur" ibid. XIV.3.

12 "non tunc res ipsae sensibus externis erunt proponendae, sed potius compendiosae quaedam illarum figurae" ibid. XII.11.

constants. All beings thus appear as mathematical signs that can be handled abstractly by algebra, that is, by a mathematics of general magnitudes representing phenomena of all kinds.

Thus we return to the first aspect mentioned above, namely, the development of mathematics itself since the seventeenth century. Decisive here is the discovery, or rather, the casting of Newton's laws of motion which, in line with Cartesian *Rules*, admit a mathematical formulation. Celestial motion provided the paradigm of physical motion whose regularity allowed scientists such as Galileo, Kepler and Newton to finally arrive at a mathematical formulation for celestial motion which adequately accounted for the empirically observed motions of the planets in particular. This mathematical formulation was then generalized to motions of all kinds of all physical bodies, including on Earth.

7. Time mathematized as a real, continuous variable

To achieve a mathematical formulation of motion, however, time could no longer be conceived in the Aristotelean way as a counting number lifted off movement, but had to become a *continuous magnitude*, for motion itself was conceived as a continuous change of position through time. The velocity of a physical body was conceived as the rate of change of position with respect to time. To attain a calculable mathematical formulation, both position and time had to be mathematized as continuous magnitudes. Position represented no great problems, for three dimensional Euclidean geometry allowed position to be accounted for by three numerical co-ordinates which had to be regarded as continuous. Similarly, and notwithstanding the discrete, countable nature of clock-time, the flow of time became the continuous temporal medium in which motion took place. Mathematical space-time is accounted for by a quadruple of four real, continuous co-ordinates (x, y, z, t), and this conception is employed even today in advanced theoretical physics.

For time to become a continuous mathematical variable it must be conceived as a continuum of now-instants, for mathematics deals only with magnitudes that are present, and what is present with regard to time

is the now. This time composed of a continuum of now-instants can be represented mathematically as the continuous variable, t, for the present moment passing through presence. Likewise, and as a consequence, the velocity of a physical body in motion can only be mathematized in an equation in which velocity is represented by the instantaneous velocity, v, at the present now-point of time, t. But, since (average) velocity is the time taken to cover a distance, how is instantaneous velocity to be dealt with mathematically? Clearly, no distance at all can be covered in a present instant of time. Zeno's arrow is frozen at each instant. Newton's and Leibniz's solution to this problem was to radicalize approximation methods of the Greek mathematician Archimedes. The motion of a physical body over a small distance during a small, finite lapse of time from now to now, is considered, and the average velocity formulated as the quotient of the two differences. To attain the simultaneous velocity, both the small distance and the small time interval are allowed to approach zero, the limit of the quotient of distance over time thus becoming the instantaneous velocity. The distance and the time interval become infinitesimally small, and a calculus with infinitesimals is born. Velocity at a now-instant in a given direction is the infinitesimal differential of change of position with respect to time, dx/dt, and acceleration is the second derivative.

8. The irrational reals in the continuum as forever absent

The problem with this mathematical conception of instantaneous velocity or the instantaneous rate of change of any other variable with respect to time is that mathematics did not have any way of conceiving the infinitesimals, which are non-zero magnitudes smaller than any real number. The infinitesimals are both within the real continuum and also outside it. This weakness in the foundations of mathematics was finally remedied only in the nineteenth century with the work by Cauchy and Weierstrass on mathematical limits and with Richard Dedekind's formulation of the real numbers as cuts in the rational numbers. Infinitesimals are infinite, countable sequences of numbers that approach the limit of zero without every reaching it. And an irrational, real

number can be regarded as an infinite, countable sequence of rational numbers approaching a non-rational limit. Thus, an irrational, real number can only be *approached* by an infinite counting process that gets as close as you like to it without ever reaching this limit. This implies that an irrational real number can only be conceived as a *counting movement* toward that can never be made present as a logical, computable ratio of natural counting numbers.

An irrational real number is forever absent from the infinite series of rationals approaching it in a counting movement. The irrationality of an irrational real number could therefore be said to consist in its *being never present, but forever arriving, forever heralded by the endless row of rational numbers announcing its arrival*. The irrationals fulfil the illogical condition of the Aristotelean ontology of movement in general as a twofold of presence and absence. They are illogical because they can never be brought to a standing presence by the rationals. Otherwise they can only be symbolized by algebraic symbols symbolizing numbers that are forever absent and beyond the grasp of a calling to presence by the logos in a definite rational number amenable to arithmetic calculation.

Moreover, this movement of counting infinitely through a rational sequence toward an irrational limit takes place within the continuum of real numbers, so that each step from one rational number to the next must pass through an infinity of irrational real numbers. The movement of rational counting itself requires the medium of the real continuum, which is largely irrational. The continuum of real numbers can be imagined geometrically as an endless continuous line. It is geometrical figure that contours real, physical bodies, so the name ‘real’ for the real numbers is well-chosen. On the other hand, however, only rational numbers can actually be calculated to obtain a definite arithmetic number that is a kind of logos as the result of a calculating λογισμός.

9. The incalculable, indeterminate quivering of all physical beings

What can we infer from this for the being of digital beings? A digital being is, in the first place, a finite sequence of binary code, consisting perhaps of billions and billions of bits, that is interpreted and calculated by the appropriate hardware in sequences of nested algorithms to bring about a foreseen effect. As binary code, i.e. a string of zeroes and ones, a digital being is nothing other than a finite rational number, whereas even a single irrational real number is a countably infinite string of bits[13] and therefore never can be inscribed logically-digitally. And yet, this binary code, interpreted as commands to be processed by a digital processor, brings forth change and movement in the real world of real, physical beings. A digital being can only represent the real world in terms of binary bits, which are logical, rational, computable numbers that always must miss the irrational continuum of the real.

For example, a computer-controlled robot on a production line can bring the robot's arm into a precisely precalculated position, which is always a rational number or an n-tuple thereof. The robot's arm, however, will always be in a real, physical position, no matter how accurate the rational position calculated by the computer is. There is therefore always an *indeterminacy* in the computer-calculated position, a certain *quivering* between a rational position and an infinity of irrational, but real positions. An irrational, real position can never be calculated by a computer, but only approximated, only approached. This signals the *ontological* limit to the calculability of physical reality for mathematical science. It is not an experimental result, but is obtained from phenomenological, ontological considerations. We must conclude: *physical reality is irrational*.

What does this imply for the understanding of being as standing presence? The standing presence of being is a temporal determination

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13 If, following Cantor, Aleph is the symbol for the countable infinity of the natural numbers, the smallest infinite number, then the infinity of the real continuum of numbers is 2 to the exponent of Aleph and the real continuum, in binary representation, is the set of all countably infinite strings of bits.

that goes hand in hand with the understanding of time as composed of a continuum of now-instants. According to the ontology of standing presence, a physical body assumes a definite position at a definite instant of time. In mathematical physics since the beginning of the modern age, the position and motion of physical bodies become calculable, but only by developing a mathematics of the continuum of real numbers that allows also the calculation of velocity and acceleration as infinitesimal differentiations of position with respect to the real, continuous variable, t. An irrational, real instant of time or an irrational, real position, however, can never by precisely calculated, but only approached by rational approximation. Insofar, a phenomenological interpretation of the calculability of the real position of physical bodies by means of the infinitesimal calculus shows that there is *no definite position of a physical body at time, t,* but only ever an *indeterminate quivering* of it between a *here-and-now* and an incalculable infinity of irrational *there-and-thens*.

Since the mathematical access to being is generalized to *all* properties insofar as they are represented quantitatively by magnitudes, changes of all kinds in physical beings can be conceived as movements of a variable with respect to the one-dimensional, real, continuous variable, t, that is always essentially both rational and irrational, standing and quivering. The state of a real physical being, however, can only be calculated from real, rational data as a countable *rational* number. *Hence the state of any real physical being is always an indeterminate quivering around a rationally calculable state. Physical reality, even on a banal macroscopic level, therefore always exceeds what can be logically, mathematically, rationally calculated.* This holds true all the more for those physical beings — ourselves — whose essential hallmark is spontaneous, *free* movement.

Let me end therefore with a quote from Goethe: “Es waren verständige, geistreiche, lebhafte Menschen, die wohl einsahen, daß die Summe unserer Existenz, durch Vernunft dividiert, niemals rein aufgehe, sondern daß immer ein wunderlicher Bruch übrig bleibe.” (“They were rational, clever, lively people who saw very well that the sum of our existence, divided by reason, never goes evenly, but always

leaves the remainder of a queer fraction." *Wilhelm Meisters Lehrjahre* 4. Buch 18. Kap.).